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# Towards sustainable mining (Part I): Valuing investment opportunities in the mining sector

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ABSTRACT

Reliance on classical valuation methods such as the net present value (NPV) has often resulted in controversial asset valuations in the mining sector. An alternative method, termed decoupled net present value (DNPV), is used to evaluate mining investment opportunities. The proposed valuation method decouples the time value of money from the risk associated with the project providing a consistent valuation method free from the problems typically associated with the application of NPV. Market risks (e.g., commodity prices, foreign exchange) can be systematically combined with non-market risks (e.g., effect on operations of climate change and/or large earthquakes). More importantly, DNPV allows seamless integration of project risk assessment performed by technical experts and risk management implemented by business executives into the financial evaluation of the project. A simplified mining investment project is analyzed using traditional techniques and compared with the proposed DNPV. The example includes a discussion about how valuation is affected by climate change and earthquake risks, and how investment in resilience and adaption can be incorporated in the proposed analysis.

## 1. Introduction

The most popular valuation methods used in the mining industry are the net present value (NPV) technique followed by its close relative the internal rate of return (IRR). These methods consist of reducing future cash flows by a single factor that grows exponentially with time. This factor is known as the risk adjusted discount rate (RADR) because the effect of time is adjusted for risk. The main problem of combining time value of money (represented by the risk free rate) and risk in a single factor when calculating the NPV of an investment is that it artificially makes the value of cash flows that occur far in the future negligible and overemphasizes the value of earlier cash flows. Thus, the results of such an analysis can be misleading, steering corporations to adopt and government entities to accept design and operation decisions that can be detrimental to society and shareholders alike in the long term.[1] For mining investors, NPV methodologies produce high volatility in the valuation of long lived mines (30–40 years) which are comparable to the volatility associated to commodity spot prices. Moreover, the overreliance of NPV methodologies makes it nearly impossible to justify climate change reliance and adaptation investments to improve mining facility chances to withstand the effect of future significant weather events related to climate change.

Although the shortcomings of the NPV methods have been widely recognized by many industry experts (e.g., Salahor, 1998; Laughton et al., 2000; Samis et al., 2006; Guj and Garzon, 2007; Hawas and Cifuentes, 2016), and alternatives proposed, NPV is still by far the valuation method of choice. Recently, a valuation method to assess the value of long-term infrastructure projects was introduced (Espinoza and Morris, 2013; Espinoza, 2014). The proposed method, termed decoupled net present value (DNPV), addresses many of the shortcomings of the NPV method while retaining the simplicity in its presentation that has led to its popularity. The DNPV methodology consists of evaluating each of the key risks associated with a project and calculating a synthetic insurance premium for each that would be demanded by a risk neutral insurance company (if such an insurance product were actually available). This hypothetical insurance policy premium, designed to protect the project's cash flows from a shortfall below the expected values in the event of an adverse outcome of a given risk (e.g., reduction of revenues due to commodity prices volatility, increase in expenses due to technical difficulties), is termed the cost of

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[1] The comments voiced by Dean Gehring (President and CEO of Rio Tinto Minerals) at the 2015 SME Conference in Denver, Colorado reinforce this point: "*I think we're going to see how mines are valued change. The way that we've all been valuing mines for years now is basically on net present value-type analysis, which is a very, very common understanding in a Western culture. Well, there's a lot of cultures that don't get that, and, in fact, to them, its value destroying, and I think that as we go forward, we're not always going to design mines around net present value. There'll be something else that we look at that tells us what really is the true value of that operation.*"

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risk. The cost of risk is subtracted from the project's expected cash flows. The subtraction of all the relevant synthetic insurance premiums leaves the remaining project cash flows virtually riskless and the DNPV valuation for the project can then be obtained by discounting the time value of the remaining riskless cash flows using the risk free rate. In some instance such insurance instruments are available in the market place and therefore there are observable prices available (e.g., at-the-money put options for commodities or specially structured insurance policies and/or financial guarantees) that can be used in the DNPV calculation. In most practical applications, such financial instruments are not available for portions (or any) of the period under consideration so the risk of the project becoming unfavorable (i.e., the revenues being lower and/or the associated expenditures being higher than anticipated) is borne by the investor. Considering risks as costs that affect cash flows is a more natural progression of a well-established business practice of buying/selling insurance products to obtain/provide protection against insurable risks. For those risks that have not been transferred to an insurance company and/or hedge provider, the investor becomes the risk bearer against unfavorable outcomes and the estimated cost of risk their compensation for taking on such risks.

In summary, the principal feature of DNPV is the decoupling of the time value of money and risk, which allows discounting future cash flows using the risk-free rate while accounting for risk as a cost to the project. This feature is paramount for valuing long-term assets as well as liabilities (e.g., asset retirement obligations, climate change resilience and adaptation measures) as these are not reduced to negligible values by the process of discounting by an artificially high RADR. The use DNPV can facilitate the use of the captive insurance concept to manage risk for large mining conglomerates when assessing mining project assets, which is the focus of this Part I dissertation on mining sustainability. Valuation of long-term liabilities associated with mining activities, which must go hand in hand with asset valuation, is the subject of a Part II companion paper (Espinoza and Morris, 2017).

## 2. The perils of using risk adjusted discount rates

Common to most mining valuation analysis is the selection of a constant RADR to calculate the NPV of an investment. If the selected RADR is greater than the project's currency risk-free rate, then the discount rate has been adjusted for risk. The seemingly innocuous assumption of using a constant RADR throughout the investment period can have a significant effect in investment decisions particularly for long term investments with long-term future liabilities such as those of mining.

Risks that can affect investment cash flows can be from many different sources and evolve over time in many different ways. These risks are typically classified in the literature as: (1) systematic (i.e., priced, non-idiosyncratic, non-diversifiable, public, market); and (2) non-systematic (i.e., unpriced, idiosyncratic, diversifiable, private, non-market). Systematic risk is associated with the type of asset to be created (e.g., commodity prices for mining projects). Non-systematic risk is a project specific risk and could be technical (e.g., the amount of ore mineral available in a geological formation and its grade) as well as non-technical (e.g., changes in the local tax code, changes in environmental regulations). The classical text book expression for calculating the NPV of an investment considering all the associated risks listed above using discrete cash flows is given by Eq. (1):

$$NPV = \sum_{t=0}^{T} \frac{\widetilde{C}_t}{(1+r)^t} \tag{1}$$

where T is the maturity (i.e., investment period), $\widetilde{C}_t$ is the expected value of a stream of uncertain future net cash flows ($C_t$), and r is RADR that lumps time value of money and the risks described above. The approximate continuous representation of NPV as a function of time is given by Eq. (2):

$$\frac{\widetilde{C}_t}{(1+r)^t} \approx \widetilde{C}_t e^{-rt} \tag{2}$$

The classical equation to estimate the discount rate to account for market risk is given by the Capital Asset Pricing Model (CAPM) in Eq. (3):

$$r = r_f + (r_m - r_f)\beta \tag{3}$$

where $r_m$ is the expected rate of return of the overall market (e.g., the S & P500 stock index); $r_f$ is the risk-free rate; and β (i.e., the company beta) is a parameter that measures the systematic risk of the asset relative to the market, and the difference ($r_m$-$r_f$) is known as the systematic risk premium ($r_p$). Consistent with Robichek and Myers (1966), the time value of money is accounted for by the first term (i.e., the risk-free rate) in Eq. (3) whereas systematic risk is accounted for by the second term. Although modern portfolio theory predicates that investors should not demand a risk premium for non-systematic risks, such a predicament is only valid for sufficiently liquid securities that can be traded in the open market where elimination of non-systematic risk can be easily achieved through diversification. For investment in real projects/assets, the average investor would typically demand compensation to take on non-systematic risks as the amount invested and the cost of analyzing multiple investment opportunities can be significant. Examples of investors demand for additional compensation to account for nonsystematic risk abound. For instance, investors typically add a country risk premium (a theoretically diversifiable risk) when evaluating mining investing opportunities. Similarly, in the biotech industry, the selected discount rates are affected by clinical success rates.

To account for non-systematic risks posed by one-off projects, an additional risk premium ($r_{ns}$) is included to lump several non-systematic risks together (e.g., Samis et al., 2006). Hence, Eq. (3) can be modified simply as (Eq. (4)):

$$r = r_f + r_s + r_{ns} \tag{4}$$

where $r_s = (r_m - r_f)\beta$ represents the systematic (i.e., market). Congruent with systematic risks, a compensation for non-systematic risk takes the form of an additional risk premium added to the risk free rate as shown in Eq. (4), implicitly assuming that systematic and non-systematic risks are governed by the same stochastic processes.

Although this simple approximation is consistent with popular representations of market risk and is easy to implement in discounted cash flow models, its impacts can be significant because of the sensitivity of NPV to the selection of the discount rate, particularly for long term projects. Using the continuous representation of NPV on the right-hand side of Eq. (2) to explore the influence on the project NPV of the apparently innocuous simplification of adding risk premiums to the risk-free rate to account for systematic and non-systematic (i.e., market and non-market) risks, the value of the investment can be expressed as (Eq. (5)):

$$C_t e^{-rt} = C_t e^{-(r_f + r_s + r_{ns})t} = C_t e^{-r_f t} e^{-r_s t} e^{-r_{ns} t} \tag{5}$$

Or alternatively as (Eq. (6)):

$$NPV(C_t, r) = NPV(C_t, r_f)\, F_s\, F_{ns} \tag{6}$$

where NPV($C_t$, $r_f$) represents the time value of money and $F_s = e^{-r_s t}$ and $F_{ns} = e^{-r_{ns} t}$ represent the risks reduction factors that vary from 1 to 0 (Fig. 1) and account for systematic (i.e., market) and non-systematic (i.e., non-market) risks, respectively. Risk reduction factors equal to 1 indicate that there is either no risk (i.e., $r_s$ = $r_{ns}$ =0) or that time $t$=0. Risk reduction factors equal to 0 indicate that risks are infinite (i.e., $r_s$ = $r_{ns}$ =∞) or that time $t$=∞. Thus, the project NPV can be interpreted as the cash flow at time t discounted using the risk free rate to account for the time value of money and further adjusted (reduced) to account for market risk ($F_s$) and non-market (non-systematic) risks ($F_{ns}$). It follows from Eq. (6) that, independent of the actual stochastic

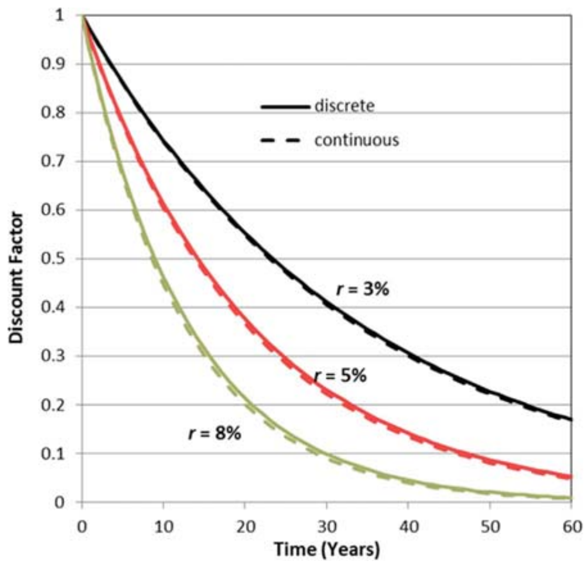


**Fig. 1.** Risk Reduction Factor Variation with Time.

processes that best describe the cash flows variation with time, adjusting the risk-free discount rate for risk (i.e., adding risk premiums) implies that, regardless of the risk source, risk factors are always represented by exponentially decaying functions of time.[2]

As discussed previously, this assumption is not accurate except in a few particular cases (Bhattacharya, 1978; Halliwell, 2001 and 2011; Giacotto, 2007) and brings excessive volatility to the valuation of long term assets. Furthermore, it follows from Eq. (6) that any error in the estimate of the discount rate will have the same functional form as $F_s$ (or $F_{ns}$) and thus will decay exponentially with time. This particular feature (i.e., exponential decay) indicates that the magnitude of this simple assumption is not trivial, particularly for long-term projects with lagging long-term liabilities such as mining projects. Depending upon the magnitude of the assumed discount rate, the cumulative error can be significant. As examples, Fig. 1 shows risk reduction factor curves for typical RADRs (3%, 5% and 8%) for a period of 60 years.

Table 1 shows specific risk reduction factors for 10-yr, 20-yr, 30-yr and 60-yr periods. Over a 60-year period, the risk reduction factors vary from 17% to 1%. Thus, assuming 8% instead of 3% would result in a 95% cumulative error over a 60-year period, rendering the present value of a $1 billion liability sixty years down the road at $10 million rather than $170 million. This artificial reduction of long-term liabilities not only poses a looming financial liability to future generation of stakeholders but also misleads investor behavior and mining operations because the mining asset is valued unrealistically high. For the same reasons, the future profitability of mining assets can be severely undermined by the use of inappropriate risk premiums for the project. For example, the present value of a $10 million constant cash flow for the next 60 years increases from $123 million using a RADR of 8% to $276 million using a RADR of 3%, a difference of over 120%. Such distortions make investments in climate change resilience and adaption hard to justify as expenses are incurred today are magnified and the benefits of reducing future contingency liabilities neglected.

The examples above illustrate the long-term magnitude of the valuation differences caused by a seemingly innocuous five-point difference in the selected RADR. Unfortunately, errors of this magnitude in selection of risk premiums are not uncommon, even in cases where risks appear well understood and a significant amount of data is available. For instance, Fama and French (1997) showed that the use of

[2] This is another issue associated with RADR. Increasing the RADR to account for market or non-market risk makes the impact of negative cash flows (e.g. losses, expenditures, contingency liabilities) that are far in the future artificially negligible.

**Table 1**
Reduction Factors.

| RADR | Mineral Extraction Period | | | |
|---|---|---|---|---|
| | 10-yr | 20-yr | 30-yr | 60-yr |
| 3% | 74.4% | 55.4% | 41.2% | 17.0% |
| 5% | 61.4% | 37.7% | 23.1% | 5.4% |
| 8% | 46.3% | 21.5% | 9.9% | 1.0% |

expressions similar to Eq. (3) to estimate an industry wide cost of equity for market risk only can be very imprecise, such that standard errors of more than 3% per year are common. This finding does not bode well for the cost of equity of individual firms (let alone specific projects) as errors in estimates of cost of equity are likely to be significantly larger when non-systematic risks are accounted for. Because of the errors in the selection of the discount rate and the propagation of these errors with time, the present value of cash flows from long-term investment projects are likely to be divergent from a properly risk adjusted valuation. Although modeling risk as an exponential decay function makes it mathematically more tractable allowing for the derivation of closed form solutions for simplified cases and the straightforward application of NPV, mathematical expedience and convenience are not valid reasons to assume that all project risks can be represented by exponential functions. Unfortunately, attempts to address this issue through parameterization of the discount rate coupled with Monte Carlo simulation will not correct the problem associated with NPV as the results are strongly influenced by the exponential decay function affecting the cash flows.

The classical NPV method as originally introduced is a top-down approach that resulted from the process of acquiring capital in the form of equity and debt and mandating that all projects must earn the firm weighted average cost of capital (WACC). Essentially, the discount rate in the classical NPV is more concerned with the sources of funding than the project itself (i.e., it is exogenous to the project) and represents a demand of equity investors and debt holders. Consistent with Eq. (3), WACC is used as a proxy for the firm's average investment risk. If a project is deemed to have a risk profile different from the firm's overall risk, business executives heuristically modify (i.e., increase) the WACC to account for the project's idiosyncratic risk. However, the continued widespread practice of heuristically adjusting (i.e., increase) the discount rate to account for actual or perceive additional idiosyncratic project risk has resulted in the deep-rooted practice of simply increasing the RADR without attempting to correlate it to the project actual risks. However, because the effect of these adjustments increase exponentially, initial errors in the selection of this parameter are magnified as the duration of the project increases. Although beyond the scope of this paper, and discussed in detail in the Part II companion paper, financial difficulties will be compounded should mine closure, reclamation, and post-reclamation care (PRC) costs increase above originally budgeted amounts, particularly as these costs occur long after the mine has stopped generating any revenues. Therefore, to accrue funds more accurately during the revenue-generating life of a mining operation such that environmentally sound reclamation and PRC programs can be implemented, a valuation procedure that does not combine the time value of money and risk is critical.

## 3. Separation of risk from the time value of money

From the discussion above, it is clear that the limited validity of an exponentially decaying function to represent all risk classes, regardless of their source, makes it difficult to perform a meaningful valuation of long-term projects. To adopt sustainable mining practices (including climate change resilience and adaptation measures), more accurate valuation methods that separate the time value of money and risk are needed. The need for such methods for valuing long term projects has been recognized for some time, with several attempts made to provide solutions. The Certainty Equivalent Method (CEM) proposed by Robichek and Myers (1966) represents the first attempt. Despite its robustness, CEM has not been widely applied in academia or practice because of the difficulties of obtaining the parameters necessary to define investors' risk preferences. Myers (1977) then connected the concept of financial options to value growth options embedded in potential corporate investments and introduced the real option valuation (ROV) method, a promissory new valuation method that could address some of the drawbacks associated with NPV. In subsequent work, Myers (1984) translated the logic of financial options to address issues of capital budgeting and strategic planning decisions and considered an extension of financial option pricing models to the valuation of nonfinancial assets (i.e., projects). Since its publication, ROV has been touted as the valuation tool that can take into account not only the changing nature of risk but also the managerial flexibility embedded in investing in real projects. ROV account for the time value of money by discounting the cash flows using the risk free rate, effectively separating the time value of money from risk. Brennan and Schwartz (1985) were the first to use ROV for valuing investments in natural resources such as mining. Since then, numerous studies using ROV to evaluate investment in natural resources have been published (e.g., Paddock et al., 1988; Smith and Nau, 1995; Moel and Tufano, 1999; Cortazar et al., 2001). However, most of these are focused on capturing the value of management's flexibility to expand, contract, defer and/or abandon projects, with little emphasis on the ability of ROV to model the changing nature of risk even for cases where there is little to no flexibility afforded. In some cases, Monte Carlo simulation is combined with ROV to analysis complex mining investment propositions (Samis and Davies, 2014). However, because of the mathematical complexity of ROV, most applications remain of academic value, are difficult to implement in practice and, more importantly, yield results that are difficult to convey to decision makers (Espinoza, 2011).

Taking advantage of the fact that options adjust future cash flows for risk such that they can be discounted using the risk free rate, Salahor (1998) and Laughton (1998a) introduced a concept termed Modern Asset Pricing (MAP) to value investments in the oil and natural gas industry. MAP is essentially derived from the work performed by Black and Scholes (1973), and uses information from future commodity prices to estimate the value of future cash flows and adjust for the risk associated to the volatility of commodity prices. To demonstrate the applicability of MAP, standard financial models typically found in discounted cash flow (DCF) analysis were used for several mining investment opportunities (Guj and Garzon, 2007; Shafiee et al., 2009; Samis and Davies, 2014). Although MAP focuses on risk adjusting cash flows, like ROV, it can also be used to model managerial flexibility (Laughton, 1998a and 1998b). Hence, in theory MAP and ROV concepts are similar and these methods could be used interchangeably. However, in this paper, the term MAP is used refer to the process introduced by Salahor (1998) and further promoted by Laughton et al. (2000) as a valuation tool for projects with little to no managerial flexibility in which the only risk accounted for is market risk (e.g., volatility of future commodity prices or foreign currency exchange rates). Although MAP constitutes a good attempt to rid the industry of problems associated with NPV and provides a method that is relatively easy to implement, there are several drawbacks with using MAP. Most significantly, as presented, it only addresses market risk and offers no provisions for including non-market risks. In some cases, non-market risk can be as, or more, important than market risk in the assessment of the value of a project.

To allow the combination of market and non-market risks in a consistent manner, Espinoza and Morris (2013) introduced the synthetic insurance concept which was used to quantify the risk of long-term infrastructure investments (not the investors' risk preference) in monetary terms and proposed the decoupled net present value (DNPV) method. DNPV is based on the CEM idea of separating risk from the time value of money. The DNPV method considers all risks as "costs" to the project (similar to paying for an insurance policy) such that the price of risk is subtracted from the potential revenues (if the risk is associated with the revenues) or added to the cost (if the risk is associated with the cost). Considering risk as a cost item to the project is more in line with the well-established business practice of buying insurance products to protect against certain insurable risks as well as with the insurance concept utilized to describe call options (Galai, 1977).[3] Following this analogy, DNPV essentially describes uncertain cash flows by three components: (i) the expected monetario value of the cash flows per each selected time period ($X$); (ii) the downside value ($D$); (iii) and the upside value ($U$). Independently of the shape of the probability density function, it can be demonstrated that $U = -D$ (i.e., selling the upside is the same as insuring the downside). What DNPV does is to explicitly quantify $D$ and to assume that the left-hand side of the distribution is transferred to a hypothetical put writer while implicitly keeping the upside $U$ so equilibrium is maintained. Thus, the value of the project minus the value of the downside ($X$-$D$) has essentially the same profit profile of a call option. The value of the investment is then calculated by discounting the cash flows reduced by risk ($X$-$D$) at the risk-free rate because risks have already been accounted for separately as a cost to the project. Although it may be tempting to explicitly add the upside, it follows from this discussion that adding the upside would cancel the downside and hence the expected cash flow would be the same as the original values and could not be discounted at the risk free rate. Practical illustrations of the application of DNPV for renewable energy investments can be found in the literature (e.g., Espinoza and Rojo, 2015; Humpert, 2016).

## 4. Valuation of a mining investment: an example

To illustrate how the DNPV method can be applied to mining investment opportunities and how the synthetic insurance for specific risks can be calculated, an example valuation of a mining investment project is provided. Detailed demonstration of the process for calculating the different insurance premiums used in the example are beyond the scope of this paper but are provided by Espinoza (2014). The example used is of a greenfield copper mine project presented by Samis et al. (2003). First, the valuation example as originally presented in 2003 using the information available at that time is described. Next, the mining investment opportunity is evaluated using the NPV method and MAP analysis, as proposed by Samis et al. (2003). Finally, to illustrate the power of the proposed DNPV methodology, hypothetical climate change resilience and adaptation risks are incorporated in the valuation of the mining asset.

### 4.1. Description of valuation example

The example project is a copper mine in a foreign country with estimated copper ore reserves of approximately 400 million tonnes with average grade of 0.5%. The total development cost was estimated at $600 million in 2003 currency.[4] The commodity spot price ($S$), long-term expected spot price ($S^*$) and foreign exchange rate (FOREX) in 2003 were $0.8/lb, $0.85/lb, and $2 per foreign currency unit (F), respectively. A description of each of these parameters is provided in Appendix A. The commodity risk was assumed to follow a mean reverting log normal process with an assumed annual volatility ($\sigma$) of 20%, a median growth ($\alpha$) of 0% and a mean reverting factor ($\gamma$) of

[3] Galai (1977) showed that call options can be viewed as a package of an insurance product and an asset.

[4] The example in Samis et al. (2003) is described in terms of Foreign Monetary Units (FMU) where the mine is located and domestic monetary units (DMU) where the currency of the equity's home country is located. In this paper, the DMU is assumed to be the dollar currency ($) and the FMU is denoted by F.

0.231. As in the original example, project cash flows were assumed to be subject to two market risks only: currency risk and commodity price risk. Equations describing the costs of commodity and currency risk are provided in Appendix B: Eq. (B.9) together with the above parameters are used to estimate the standard deviation as a function of time. The additional return per unit of volatility ($\lambda$) was 0.4. FOREX was assumed to follow a non-reverting log normal process (i.e., geometric Brownian motion) with annual volatility of 40%. The nominal riskless rate for the host country was 12.5% with an assumed inflation of 7.5%. Similarly, the investor's domestic nominal riskless rate was 3.0% with an inflation rate of 1.5% (i.e., the real rate is 1.5%).

The results obtained by Samis et al. (2003) are replicated in Table 2 (albeit with somewhat different organization to facilitate the subsequent discussion) using the information summarized above. Forward prices (Line 05) are calculated using Eqs. (A.2)–(A.6). FOREX risk factors are presented in Line 06. To estimate the expected operating revenues, only the expected commodity prices (Line 03) and the expected copper production (Line 07) were needed. The estimated copper production rate was 20.4 million tonnes (224.7 million pounds of copper) per year.

The project output is settled in dollars whereas expenses are incurred in both currencies. To facilitate the presentation, the uninflated OPEX and CAPEX (in real 2003 foreign and domestic currency) and operating revenues are shown in the Table 2. The annual (real) operating revenues (Line 08) is calculated as the product of the commodity spot price (Line 03) times the annual copper production (Line 07)[5] The uninflated OPEX was estimated at \$0.4 per pound of copper which resulted in \$89.87 (i.e., 224.7×0.4) million per year (Line 10). As shown in Lines 11 and 12, respectively, 25% of the OPEX is incurred in domestic currency (i.e., \$22.47 million) and the remainder in foreign currency (i.e., 0.75×\$89.7/2= F33.7 million). The project required four years to develop. The corresponding CAPEX for the first four years in real foreign and domestic currency is shown in Lines 14 and 15, respectively. Similarly, closure costs are assumed to be incurred in Year 24 (the last year of production) in both currencies and are shown in Lines 16 and 17, respectively. Total CAPEX is shown in Line 13.

### 4.2. NPV analysis

To calculate the NPV of the proposed investment, Samis et al. (2003) calculated revenues using forecasted expected spot prices unadjusted for risk. To account for currency risk, expenses in denominated foreign currency were converted to nominal domestic currency (dollars) using futures.[6] Thus, operating revenues (in dollars) were obtained by multiplying forecasted mean spot prices (Line 03 in Table 2) by expected copper production (Line 07). Because forecasted mean prices are in 2003 currency, the calculated operating revenues are also real. Nominal OPEX and CAPEX (Lines 20 and 22, respectively) in domestic currency were obtained by inflating the expenses using the domestic inflation rate of 1.5%. Similarly, nominal OPEX and CAPEX in foreign currency (Lines 19 and 21, respectively) were obtained by inflating the expenses using the foreign inflation rate of 7.5%. Expenditures in foreign currency were then transformed to dollar denominated futures by adjusting the inflated amounts by the FOREX factor (Line 06). The total cost (the sum of Lines 19 through 22) in nominal dollars is presented in Line 18. The nominal expected operating cash flows (Line 24) is simply the difference between operating revenues (Line 09) and total costs (Line 18). As discussed above, currency risk is accounted for by the use of futures. Thus, the assumed RADR of 10% is mainly intended to account for commodity risk and any other risks impacting the expected project cash flows. To obtain the project discounted cash flows (Line 26), Samis et al. (2003) adjusted the operating cash flows (Line 24) by multiplying these quantities by the continuous representation of the risk factor (i.e., $e^{-RADR\ t}$). The calculated NPV using a RADR of 10% is \$132.18 million (i.e., sum of Line 26). The variation of currency factor to estimate currency futures contracts is presented in Line 06. As shown, the currency factor decreases with time. Note that the use of future currency prices to account for foreign exchange risk in the MAP analysis appears to be inconsistent with the notion of risk (i.e., higher risk, lower valuation). In this example, incurring all expenditures in foreign currency would make the project more valuable than if all expenditures were incurred in dollars.

### 4.3. MAP analysis

To value the project using MAP analysis, Samis et al. (2003) calculated the operating revenues using the values of futures contract prices instead of expected commodity prices. Conceptually, this is similar to assuming that future mining production has been contracted at the calculated future contract prices, thus accounting for the risk associated with commodity prices. Since prices for copper futures up to five years are typically available, it is assumed that the copper futures data for the first five years in Table 2 corresponded to actual market data available at the time, and that remaining futures prices from years 6 through 24 were forecasted using Eq. (A.6), as shown in Fig. 2. To that end, the risk adjusted operating revenues (Line 09) were calculated as the mineral risk discount factor (Line 04) times the operating revenues (Line 08). The mineral risk discount factor (Eq. (A.5)) was derived by linking expected and future prices. Risk adjusted net cash flow was calculated as risk adjusted operating revenues (Line 09) minus the total nominal expenses (Line 18), which are already risk adjusted to account for currency risk. The valuation using the MAP method (i.e., reducing the revenue cash flows by a factor and discounting the resulting cash flows at the risk free rate using the continuous form of the discount) is \$262.42 million (the sum of Line 25). Note that the valuation obtained is higher than the \$132.18 million NPV valuation (a \$262.42 million valuation would imply the use of an 8.3% RADR for the NPV methodology).

In essence, MAP analysis does not appear to introduce significant additional effort or scrutiny of risk as compared to NPV analysis but rather focuses only on a single risk factor: the volatility of commodity prices. Although MAP analysis is a step in the right direction as it promotes separation of the time value of money from risk, the method has several shortcomings that limits its effectiveness. The method essentially assumes that risk in commodity prices are accounted for by using future prices in the cash flow analysis. In theory, this approach would be correct if the only risk associated to the revenue side were due to commodity future prices and the entire future copper production shown in Table 2 is contracted at the estimated future prices thus eliminating the risk of commodity prices. However, as applied, the MAP approach may create inconsistencies depending upon whether or not actual future contracts are used. For instance, the MAP analysis does not differentiate between an investment proposition where there is purchase agreement in place and the total production is contracted at the estimated future prices and a similar investment proposition where none of the production is contracted. In both cases, the same future prices would be used but, in the first case, the investment would obviously be less risky. In the case when the risk is associated with expenditures, the use of futures contracts to account for foreign currency risks may create additional inconsistencies in the valuation analysis as discussed above. As future currency decreases with time, the value of future expenses are increasingly reduced. In addition, the MAP method does not account for non-market risks: as is the case with

[5] Note that although Table 4 of Samis et al. (2003) labels these cash flows as nominal, they represent 2003 currency (i.e., real) as changes of annual prices over time are simply the result of regression to the long-term average of \$0.85/lb expressed in real (2003) currency.

[6] Typical NPV analysis would use both revenues and expenses unadjusted for risk and account for risk for both FOREX and commodity prices in the RADR.

**Table 2**
Mining Example from Samis et al. (2003) (in millions of dollars unless indicated otherwise): NPV and MAP Analysis.

| | Time (yr) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01 | Variance (Eq. (A.2)) | | 0.032 | 0.052 | 0.065 | 0.073 | 0.078 | 0.081 | 0.083 | 0.084 | 0.087 | 0.087 | 0.087 | 0.087 |
| 02 | Median price [$/lb] (Eq. (A.3)) | | 0.810 | 0.818 | 0.825 | 0.830 | 0.834 | 0.837 | 0.840 | 0.842 | 0.850 | 0.850 | 0.850 | 0.850 |
| 03 | Mean spot price [$/lb] (Eq. (A.4)) | 0.8 | 0.823 | 0.840 | 0.852 | 0.861 | 0.867 | 0.872 | 0.875 | 0.878 | 0.887 | 0.887 | 0.887 | 0.887 |
| 04 | Mineral Risk Discount (Eq. (A.5)) | | 0.931 | 0.880 | 0.841 | 0.812 | 0.789 | 0.771 | 0.758 | 0.747 | 0.709 | 0.709 | 0.708 | 0.708 |
| 05 | Forward Price [$/lb] (Eq. (A.6)) | | 0.766 | 0.739 | 0.716 | 0.698 | 0.684 | 0.672 | 0.663 | 0.656 | 0.629 | 0.629 | 0.629 | 0.628 |
| 06 | FOREX Factor [$/F] (Eq. (A.7)) | 2.000 | 1.819 | 1.654 | 1.504 | 1.368 | 1.244 | 1.131 | 1.029 | 0.935 | 0.272 | 0.247 | 0.225 | 0.205 |
| 07 | Copper Production ($10^6$ lbs) | | | | | | **224.67** | **224.67** | **224.67** | **224.67** | **224.67** | **224.67** | **224.67** | **136.56** |
| 08 | Operating Revenues (real) | | | | | | $194.805 | $195.885 | $196.695 | $197.309 | $199.321 | $199.341 | $199.357 | $121.185 |
| 09 | Risk Adjusted Oper. Rev. (real) | | | | | | $153.667 | $151.080 | $149.020 | $147.380 | $141.359 | $141.294 | $141.243 | $85.829 |
| 10 | Total OPEX (Real $) | | | | | | $89.87 | $89.87 | $89.87 | $89.87 | $89.87 | $89.87 | $89.87 | $54.63 |
| 11 | Foreign (F) | | | | | | F33.70 | F33.70 | F33.70 | F33.70 | F33.70 | F33.70 | F33.70 | F20.48 |
| 12 | Domestic ($) | | | | | | $22.47 | $22.47 | $22.47 | $22.47 | $22.47 | $22.47 | $22.47 | $13.66 |
| 13 | Total CAPEX (Real $) | **$97.20** | **$178.25** | **$194.40** | **$113.45** | **$16.75** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $50.00 |
| 14 | Initial (F) | **F36.45** | **F66.83** | **F72.90** | **F42.53** | **F6.08** | | | | | | | | |
| 15 | Initial ($) | **$24.30** | **$44.60** | **$48.60** | **$28.40** | **$4.60** | | | | | | | | |
| 16 | Closure (F) | | | | | | | | | | | | | **F18.75** |
| 17 | Closure ($) | | | | | | | | | | | | | **$12.50** |
| 18 | Total Cost (Nominal $) | $97.20 | $176.28 | $190.16 | $109.80 | $16.10 | $85.20 | $84.36 | $83.55 | $82.77 | $75.07 | $74.66 | $74.27 | $86.05 |
| 19 | Foreign related OPEX ($) | | | | | | $60.99 | $59.78 | $58.60 | $57.43 | $44.29 | $43.41 | $42.55 | $25.35 |
| 20 | Domestic related OPEX ($) | | | | | | $24.22 | $24.58 | $24.95 | $25.33 | $30.79 | $31.25 | $31.72 | $19.57 |
| 21 | Foreign related CAPEX ($) | $72.90 | $131.00 | $140.08 | $80.10 | $11.22 | | | | | | | | $23.20 |
| 22 | Domestic related CAPEX ($) | $24.30 | $45.27 | $50.08 | $29.71 | $4.88 | | | | | | | | $17.92 |
| 23 | Risk Adjusted Net Cash Flow | −97.200 | −176.278 | −190.163 | −109.804 | −16.100 | 68.464 | 66.719 | 65.470 | 64.614 | 66.289 | 66.636 | 66.971 | −0.217 |
| 24 | Expected Operating Cash Flow | −97.200 | −176.278 | −190.163 | −109.804 | −16.100 | 109.602 | 111.523 | 113.146 | 114.542 | 124.251 | 124.682 | 125.085 | 35.139 |
| 25 | Real Options NPV | −97.200 | −171.068 | −179.089 | −100.354 | −14.280 | 58.928 | 55.728 | 53.069 | 50.827 | 35.305 | 34.441 | 33.591 | −0.106 |
| 26 | DCF NPV | −97.200 | −159.503 | −155.692 | −81.345 | −10.792 | 66.477 | 61.205 | 56.187 | 51.467 | 15.215 | 13.815 | 12.541 | 3.188 |

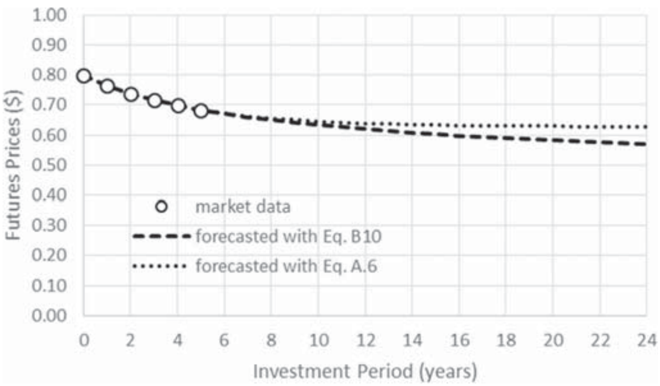


**Fig. 2.** Forecasted future commodity prices.

CAPM, MAP assumes that investors should not be compensated for taking diversifiable risk.

### 4.4. DNPV analysis

To make a direct comparison with both NPV and MAP analysis, the cash flow analysis presented in Table 2 is evaluated using DNPV considering only the two market risks discussed in the original analysis (i.e., commodity and exchange rate risk). Using Eq. (B.10) to calculate copper future prices, data for the first five years was curve-fitted and a convenience yield δ equal to 3.0% obtained. The forecasted commodity future prices for years 6 through 24 using Eqs. (A.6) and (B.10) are presented in Fig. 2 for comparison purposes only. Using the estimated convenient yield (δ) equal to 3.0% along with Eq. (B.9), at-the-money puts and calls were estimated using Eqs. (B.1)–(B.4). Assuming that currency futures follow a geometric Brownian motion (GBM) process, it can be shown that δ is equal to ($r_f$-$r$) (i.e., 9.5%).

Table 3 shows the annual revenues as well as CAPEX and OPEX in real 2003 currency along with the cost of risks associated with commodity and currency prices calculated using the equations summarized in Appendices A and B, respectively. As predicated by the DNPV methodology, regardless of the source, risk always adversely affects the value of the project (Espinoza and Morris, 2013). For the two risks analyzed, a drop in commodity prices would adversely affect the revenue side of the equation whereas an increase in the value of foreign currency against the investor's domestic currency would adversely affect expenditures (i.e., it would take more dollars to buy the same local services). Thus, for this investment opportunity, a put option would represent the cost of risk associated with a drop in commodity prices (Line 13 in Table 3) whereas a call option on the value of the foreign currency would represent the cost of risk of appreciation of the foreign currency (Line 14). The decoupled net cash flows (Line 15) are obtained by subtracting these costs of risk (Lines 13 and 14) from the net cash flows in real dollars (Line 11). Discounting Line 15 by the real domestic rate (i.e., 1.5%), the calculated DNPV is $189.78 million. An equivalent risk adjusted discount rate of 8.4% applied to cash flows from Line 11 would yield the same DNPV value.

### 4.5. Integration of physical (non-market) risks

The power of DNPV is not only its ability to model market risks such as commodity and currency risks but also its ability to evaluate the physical risks and integrate them into the financial analysis in a clear, transparent, and consistent manner. Although commodity and currency risks are important sources of risk in a mining investing project and serve as the basis of illustration herein, deeper consideration of additional risks that could affect the value of a mining investment opportunity is of course also needed to avoid significant overvaluation. A summary of typical risks that could affect mining investments is presented in Table 4.

To further illustrate the power of the DNPV method and how these risks affect the analysis, four additional risks selected from Table 4 (i.e., grade risk, OPEX risk, temporary shutdown risk due to climate change effects, and permanent shutdown risk due to a large earthquake) were included in the financial evaluation presented in Table 3. The cost of these risks (Lines 17–20 of Table 3) were computed using the equations summarized in Appendix C. As shown in Fig. 3, the present value of the total cost of risk is $1039.6 million. The percent contribution of each risk to the total cost of risk is summarized in Table 3 and depicted in Fig. 3.

The decoupled net cash flows for this expanded case are obtained by adding these additional costs to the cost of commodity and currency risks and subtracting the results from the net cash flows. Discounting Line 21 by the real domestic rate (i.e., 1.5%), the calculated DNPV is $54.56 million. An equivalent risk adjusted discount rate of 10.8% applied to cash flows from Line 11 would yield the same DNPV value. This represents an increase of 3.4% in risk premium when compared to the case where only commodity and foreign currency risk were considered.

**Table 3**
Mining Example (in millions of dollars unless indicated otherwise): DNPV Analysis.

| | Time (yr) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 21 | 22 | 23 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 01 | Copper Production ($10^6$ lbs) | | | | | | 224.67 | 224.67 | 224.67 | 224.67 | 224.67 | 224.67 | 224.67 | 136.56 |
| 02 | Operating Revenues (Real) | | | | | | $179.73 | $179.73 | $179.73 | $179.73 | $179.73 | $179.73 | $179.73 | $109.25 |
| 03 | Total OPEX (Real) – Millions | | | | | | **89.87** | **89.87** | **89.87** | **89.87** | **89.87** | **89.87** | **89.87** | **54.63** |
| 04 | Foreign (F) | | | | | | 33.700 | 33.700 | 33.700 | 33.700 | 33.700 | 33.700 | 33.700 | 20.484 |
| 05 | Domestic ($) | | | | | | 22.467 | 22.467 | 22.467 | 22.467 | 22.467 | 22.467 | 22.467 | 13.656 |
| 06 | Total CAPEX (Real) | **97.2** | **178.25** | **194.4** | **113.45** | **16.75** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **50.00** |
| 07 | Initial (F) | 36.45 | 66.825 | 72.9 | 42.525 | 6.075 | | | | | | | | |
| 08 | Initial ($) | 24.3 | 44.6 | 48.6 | 28.4 | 4.6 | | | | | | | | |
| 09 | Closure (F) | | | | | | | | | | | | | 18.75 |
| 10 | Closure ($) | | | | | | | | | | | | | 12.5 |
| 11 | Net Cash Flows (Real) | -$97.20 | -$178.25 | -$194.40 | -$113.45 | -$16.75 | $104.94 | $106.02 | $106.83 | $107.44 | $109.45 | $109.47 | $109.49 | $16.56 |
| 12 | COST OF RISK | | | | | | | | | | | | | |
| 13 | Commodity (75.0%) | | | | | | $37.37 | $39.79 | $41.62 | $41.85 | $56.49 | $57.08 | $57.62 | $35.33 |
| 14 | Currency (12.0%) | | $14.73 | $18.65 | $11.25 | $1.59 | $8.51 | $8.10 | $7.64 | $7.15 | $2.44 | $2.23 | $2.04 | $2.17 |
| 15 | Decoupled Net Cash Flows | -$97.20 | -$192.98 | -$213.05 | -$124.70 | -$18.34 | $59.12 | $58.13 | $57.34 | $56.67 | $50.52 | $50.16 | $49.83 | −20.94 |
| 16 | ADD. COST OF RISK | | | | | | | | | | | | | |
| 17 | Grade Risk (4.5%) | | | | | | $2.92 | $2.93 | $2.95 | $2.96 | $2.99 | $2.99 | $2.99 | $1.82 |
| 18 | OPEX Risk (5.3%) | | | | | | $3.49 | $3.49 | $3.49 | $3.49 | $3.49 | $3.49 | $3.49 | $2.12 |
| 19 | Temporary Shutdown (0.35%) | | | | | | $0.22 | $0.22 | $0.22 | $0.22 | $0.22 | $0.22 | $0.22 | $0.13 |
| 20 | Permanent Shutdown (2.9%) | | | | | | $3.32 | $3.18 | $3.03 | $2.88 | $0.69 | $0.51 | $0.31 | $0.12 |
| 21 | Decoupled Net Cash Flows | -$97.20 | -$192.98 | -$213.05 | -$124.70 | -$18.63 | $49.17 | $48.31 | $47.66 | $47.12 | $43.13 | $42.97 | $42.82 | -$25.13 |

**Table 4**
Risk Identification Analysis.

| Source | Parameter | Potential Risk | Risk Magnitude/Management |
|---|---|---|---|
| **Revenue** | Ore Quantity | Less than estimated from geological analysis | Moderate: risk assumed by the investor. This risk can be reduced with increased field exploration |
| | Mineral Grade | Quality of the ore less than anticipated | Moderate: risk to be assumed by investor. This risk can be reduced with filed/lab exploration |
| | Mineral prices | Less than assumed in financial analysis. | Significant: risk can be shared among different players. Portion of the production could be contracted at prevalent future prices |
| | Commodity Production | Temporary shutdown (e.g., strikes; flooding; water shortage, minor earthquake) | Moderate/high: reduce strikes potential by Implementing a program that monitors employee satisfaction, reduce flooding/earthquake damage by improving water/earthquake resilience. |
| | | Permanent shutdown (expropriation, large earthquake) | Moderate/High: Expropriation risk can be mitigated through government backed guarantees to protect against permanent shutdown; Earthquake risk can be reduced by improving the design |
| **Expenditures** | Operating Expenditures | More expensive to operate than anticipated due to unforeseen conditions (e.g., water shortage) | Medium/High: Operations can be restructured to improve efficiency and account for potential climate change. |
| | Capital Expenditures | Higher CAPEX than anticipated | Low: Obtain competitive quotes from qualified vendors. |
| | Reclamation Cost | Expenses associated to mine reclamation costing more than anticipated | High: Lack of reclamation standards may leave the door open for additional mine reclamation work |
| | Post reclamation care (PRC) | Costing more than anticipated (e.g., | High: Use well known technologies; perform well designed pilot tests |
| | Custodial Care | Taking longer than anticipated (perpetuity) | High: Generate income from closed property to cover incidentals |
| | Currency | Expenditures can be increased due a unfavorable change in the foreign exchange rate | High: Enter into futures contract agreements, purchase equipment when exchange rates are favorable |

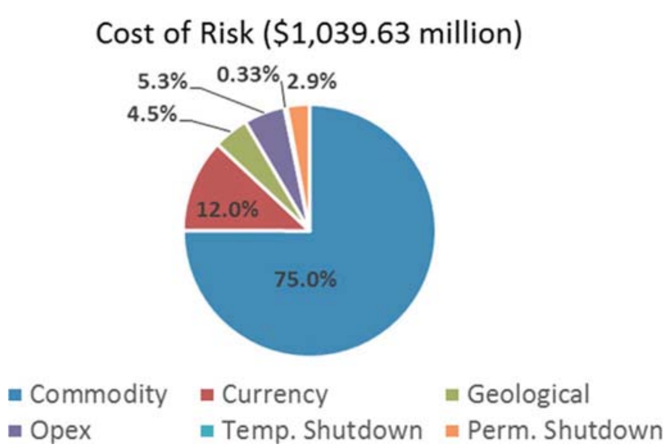


**Fig. 3.** Risk Contribution to Total Risk.

This example shows that even a small risk (i.e., probability of occurrence of 0.1%) such as permanent shutdown due to a large earthquake has a small but noticeable effect on the investment valuation (i.e., 2.9% of the total risk or $30.14 million). An increase in the likelihood of temporary shutdown due to climate change from 4% to 20% percent increases the cost of this risk from $3.6 million to $18.1 million reducing the DNPV to $40.1 million. An equivalent risk adjusted discount rate of 11.1% applied to cash flows from Line 11 would yield the same DNPV value. This represents a slight increase from in risk premium compared to the case without considering the effect of climate change. A summary of all cases analyzed is presented in Table 5.

**Table 5**
Summary of DNPV Cases Analyzed.

| Description of Risks | DNPV (millions) | Equivalent RADR |
|---|---|---|
| Commodity and currency risks only | $189.8 | 8.4% |
| Mineral grade, operation, temporary and permanent shutdown risks added | $54.6 | 10.8% |
| Climate change risk included | $40.1 | 11.1% |

Although the change in value due to climate risk is marginal for this application, this simple example shows how increased climate change risk can be incorporated in valuation of a project. For this case, managers could consider an investment in increased climate change resilience of $14.5 million (i.e., $18.1 - $3.6) or less to reduce the effects of an increase frequency of 25-yr storms on the mining operations. Similarly, the risk of permanent shutdown (i.e., $30.1 million) can be reduced to negligible values investing in mining infrastructure earthquake resilience. This investment would be considered worthwhile if the cost to improve the mining infrastructure is less than $30.1 million. Alternatively, if cost-effective insurance products are available, managers may be adopting this option instead. This constitutes a valuable tool for managers to communicate with decision makers about the need to invest on CAPEX to reduce future contingency risk of temporary shutdown and/or permanent shutdown. Crucially, DNPV provides a mechanism to quantify both technical and non-technical risks in monetary terms which can then be used to make and communicate decisions to stakeholders.

## 5. Conclusions

A promising valuation method termed DNPV is presented in the context of valuing mining investment opportunities. DNPV integrates technical risk and market risk and, regardless of the sources of risk, defines such risk as costs to the project cash flows. Revenue risks (e.g., reduced commodity prices) reduce overall revenues whereas expense risks (e.g., unfavorable foreign exchange rates) increase overall expenses. Because DNPV requires individual project risks to be identified and quantified, the effect of risk management measures (risk mitigation and risk allocation through time) can be directly evaluated.

A hypothetical example taken from a mining industry publication is used to compare the DNPV valuation with other popular valuation techniques such as NPV and MAP. The inconsistencies in the analysis in these methods are highlighted. The example also serves to clearly illustrate the negative effect of using risk adjusted discount rates to value long-term mining investments. Current valuation methods, NPV in particular, significantly discount the value of future liabilities thereby masking the true mineral extraction costs. This artificial reduction of long-term liabilities not only poses a looming financial liability to future generation of stakeholders but also misleads investor behavior and mining operations because the mining asset might be valued unrealistically high. The example presents a discussion regarding how non-market risks (e.g., climate change and earthquakes) can affect the project risk profile and the effect of these risks on the value of the investment. The example shows how investing in climate change resilience measures can be incorporated in the valuation process. Although DNPV presents a single value result, such value represents the integration of multiple scenarios described by probability density functions of each of the selected risk parameters analyzed. In addition, to facilitate communicating the results to decision makers and present the results in more familiar terms, equivalent risk adjusted discount rates can be calculated for each cased analyzed to show the effect of project risks on this parameter. This represents a significant departure from industry practice where risk adjusted discount rates are selected not calculated. Finally, although not presented in this paper, the additional value that may be obtained in a mining investing opportunity through the inclusion of operational flexibility (i.e., defer, close, expand) can be easily incorporated within the proposed framework.

## 6. Disclaimer

This research received no specific funding from any source in the public, commercial, or non-profit sectors. The opinions expressed in this article are those of the authors, and do not reflect in any way those of the institutions to which they are affiliated, including any parent entities or subsidiaries.

## Appendix A. MAP Formulation

Commodity prices were assumed to follow a reverting log-normal stochastic process given by (Salahor, 1998):

$$dS = \left[\alpha + 0.5\sigma^2 - \gamma \ln\left(\frac{S}{S^*}\right)\right]Sdt + \sigma\, S\, dz \tag{A.1}$$

where $S$ in the current commodity spot price; $S^*$ is the long-term median spot price; $\sigma$ is the short-term volatility; $\alpha$ is the short-term growth of the price medians, and $\gamma$ is the rate of reversion. For this example, the parameters $\sigma$, $\alpha$ and $\gamma$ were selected to be in agreement with Samis et al. (2003), that is 20%, 0, and 0.231, respectively.

The variance ($\text{Var}_o$), median ($M_o$), and mean ($E_o$) of the commodity spot price at time $T$ (S($T$)) are given by the following expressions:

$$Var_o[S(T)] = \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma T}) \tag{A.2}$$

$$M_o[S(T)] = S^*\left[\frac{S_o}{S^*}\exp\left(\frac{\alpha}{\gamma}(1 - e^{-\gamma\, T})\right)\right]^{\exp(-\gamma\, T)} \tag{A.3}$$

$$E_o[S(T)] = M_o[S(T)]\exp[0.5Var_o[S(T)]] \tag{A.4}$$

The commodity prices is assumed mean reverting; thus, the risk discount factor ($R_{MR}$) and the corresponding future price are given by (Salahor, 1998):

$$R_{MR} = \left[\frac{\lambda\sigma}{\gamma}\exp(1 - e^{-\gamma\, T})\right] \tag{A.5}$$

Eq. (A.4) along with (A.5) can be used to estimate the value of the commodity future price as:

$$F_t = E_o[S(T)]R_{MR} \tag{A.6}$$

The effect of the currency risk can be taken into consideration by multiplying Eq. (A.6) times a currency risk factor ($R_{FE}$) which can be calculated as (Hull, 2012, pp 115):

$$R_{FE} = \exp[(r_F - r)\, t] \tag{A.7}$$

where $r_F$ and $r$ are the foreign currency and dollar risk free rates, respectively.

## Appendix B. Commodity and currency cost of risk

To calculate the project DNPV, the downside potential for both the commodity and currency risk needs to be estimated. Eqs. (B.1) and (B.2) shown below are the classical representation of an European call/put option developed by Black and Scholes (1973) which gives an investor the right, but not the obligation, to buy/sell a stock, bond, or any other instrument ($S$) at a specified price (called the exercise price, $X$) within a specific period of time ($T$).

$$\frac{C}{S} = e^{\delta\, T}N(d_1) - \frac{X}{S}e^{r_f\, T}N(d_2) \tag{B.1}$$

$$\frac{P}{S} = \frac{X}{S}e^{r_f\, T}N(-d_2) - e^{\delta\, T}N(-d_1) \tag{B.2}$$

where $C$ and $P$ are the prices of a call option and a put option, respectively (i.e., the risk premium or price of risk) and the parameters $d_1$ and $d_2$ are calculated as:

$$d_1 = \frac{\ln(S/X) + ((r_f - \delta)T + 0.5\sigma_T^2)}{\sigma_T} \tag{B.3}$$

$$d_2 = d_1 - \sigma_T \tag{B.4}$$

$$\sigma_T = \sigma\sqrt{T} \tag{B.5}$$

where $\delta$ is the dividend (or convenience yield) paid by the stock, bond, or any other instrument; $\sigma$ is a measure of the volatility (risk) of the stock; and the operator N() is the cumulative standard normal distribution function. Eq. (B.5) represents the standard deviation of the GBM stochastic process and it grows with time (i.e., risk increases the further into the future). Eqs. (B.1)–(B.5) were derived for a GBM process of the form:

$$dS = (\mu - \delta)S\ dt + \sigma\ Sdz \tag{B.6}$$

where $\mu$ represents the stock expected growth rate. Although Eqs. (B.1)–(B.5) was derived for a GBM process, it has been shown that similar expressions can be used to model nonlinear stochastic process (Espinoza, 2011). Thus, Eq. (A.1) can be rewritten as Eq. (B.6) by setting:

$$\delta = \gamma Ln(s) \tag{B.7}$$

$$\mu = \alpha + 0.5\sigma^2 + \gamma\ \ln(S^*) \tag{B.8}$$

Eqs. (B.6)–(B.8) represent the Ornstein-Uhlenbeck mean reverting (MR) process. Different from the GMB process, in the MR process, the convenience yield is not constant but depends on the value of S. Approximate solutions for the option prices can be obtained from Eqs. (B.1)–(B.4) with the standard deviation given by Eq. (B.9) below instead of Eq. (B.5) to account for the fact that the standard deviation does not increases boundless with time:

$$\sigma_T^2 = \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma T}) \tag{B.9}$$

Thus, because future prices can be observed from the futures market and can also be extrapolated, it follows that future prices ($F_T$) can be correlated to the spot price with an at-the-money collar, that is:

$$F_T = S_o + C(K_T, \sigma_T, \delta) - P(K_T, \sigma_T, \delta) \tag{B.10}$$

where $S_o$ is the current spot price, $K_T$ is the strike price at time $T$, (equal to $S_o$), $\delta$ is the commodity convenience yield, $C$ is the value of the call, and $P$ is the value of the put given by Eqs. (B.1) and (B.2), respectively. Because future prices are either known from market data or forecasted from market data, Eq. (B.10) can be used to estimate the convenience yield.

## Appendix C. Other cost of risks

*Ore grade risk*

To account for the technical risk, the variability in copper grades is assumed to be normally distributed with a standard deviation equal to 0.02%. In this case, the downside is represented by the area to the left side of the expected value (i.e., the grades, and therefore revenues, lower than expected). The probability that the copper grades is less than 0.44% (i.e., three standard deviations less than the mean) is about 1 in 740 (0.00135). The expected downside for this copper grade is simply the center of gravity of the downside (i.e., half of the bell distribution) and it is calculated at 1.5% of the estimated revenues (Fig. C.1).

*OPEX risk*

The variability of the estimated OPEX is assumed to be represented by a triangular distribution with a minimum, most likely and a maximum of \$0.35/lb, \$0.40/lb and \$0.54/lb, respectively. In this case, the downside is represented by the area to the right side of the expected value (i.e., the

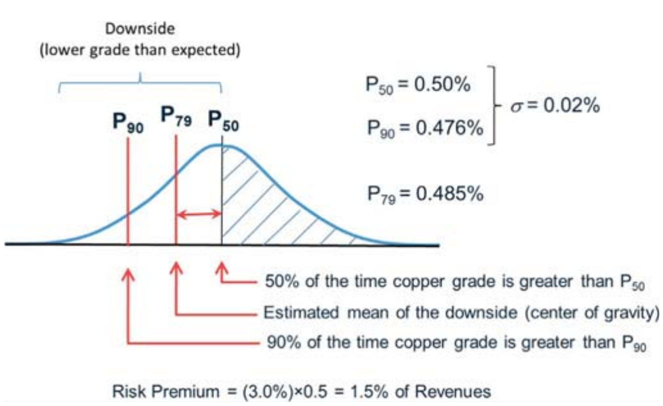


**Fig. C.1.** Copper Grade Distribution.

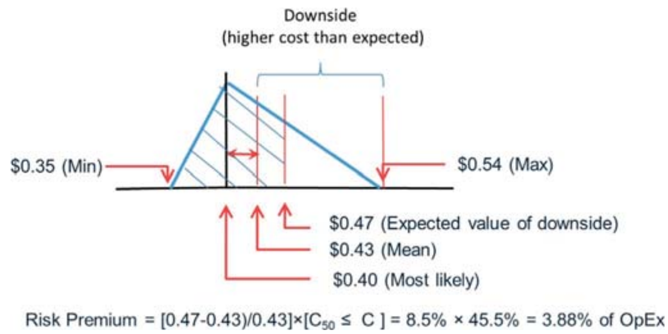


**Fig. C.2.** OPEX cost distribution.

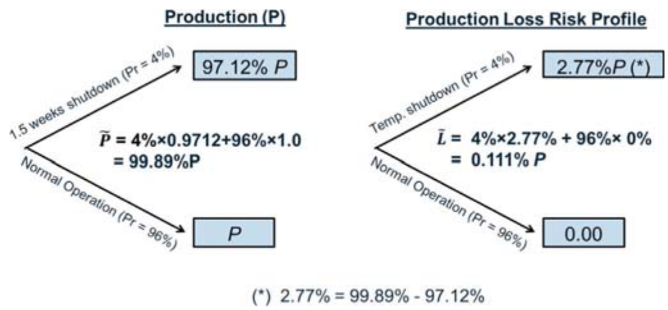


**Fig. C.3.** Temporary Shutdown Risk Profile.

OPEX greater than expected). The expected value of the OPEX for the assumed distribution is $0.43. The estimated cost of risk is calculated to be $0.02 which corresponds to 3.88% of the expected value (Fig. C.2).

*Temporary shutdown risk due to climate change effects*

Risks associated to climate change can be modeled in a consistent manner using the proposed DNPV methodology. More importantly, because risks are expressed in dollar terms, the financial effect of implementing resilience measures can be evaluated in a straight forward manner. For instance, assuming that if the mining facility is hit by a storm greater than a 25-yr storm, operations and/or production would be halted temporarily for 1.5 weeks on average due to power loss, flooding, slope failures and related incidents. This risk can be modeled as a binomial risk and does only affect the annual revenues. The frequency of a 25-yr storm is once every 25 years (i.e., the annual probability of ocurrence $P_r$ is 4%) and the average downtime of 1.5 weeks corresponds to an average production loss in any given year of 2.88% (1.5 week/52). Thus, the expected production in any given year is 99.88% (i.e., 96%×1.0+4%×0.9712) of the expected annual production (*P*). For simplicity, this risk can be modeled as a binomial event. Fig. C.3 represents this risk. As shown in this figure, the cost of risk is estimated to be approximately 0.11% of the total revenues. This risk can be considered independent and identically distributed with time. An increased storm frequency due to climate change effect can be easily incorporated in the proposed methodology. For instance, if the frequency and severity of storms is expected to increase 5 fold due to climate change effects, then the 25-yr storm events will have a probability of ocurrence $P_r$ =20%. The expected production would fall to 99.42%*P* whereas the cost fo risk would increase to 0.46% *P* (see Fig. C.4).

*Permanent shutdown risk due to a large earthquake*

A potential large earthquake risk can be modeled as a Poison process (i.e., negative jump). For this case, the earthquake is so massive that it could cripple the mine leading the owners no alternatives other than closing operations permanently, thus affecting future potential cash flows (this risk affects both revenues and expenses) from the time the event takes place onward. The methodology to estimate the cost of risk for this particular risk is described in detail by Espinoza (2014). As described, the intertemporal relationship of the cost of risk in a given year is captured by considering the remaining cash flows (RCF) from that year on discounted at a rate equal to $r_f+\Theta$ where $r_f$ is the risk free interest rate and $\Theta$ is the permanent shutdown risk factor.[7] Assuming Level I risk neutrality as defined by Espinoza (2014), the risk factor can be estimated as the expected loss of remaining revenues. It is assumed that the country where mining is taking place has a risk of a large earthquake equal to 0.01%. As in the previous case, this risk can be modeled as a binomial risk. As shown in Fig. C.5, the risk factor is simply $\Theta$=0.01% which can be used to estimate the cost of risk for each period. It follows that as time goes by, future RCF are reduced; hence, the risk associated to earthquake also decreases.

[7] Although the demonstration included in Appendix A of Espinoza (2014) assumes constant revenues throughout the period analyzed, it can be shown that the same result is obtained assuming variable revenues.

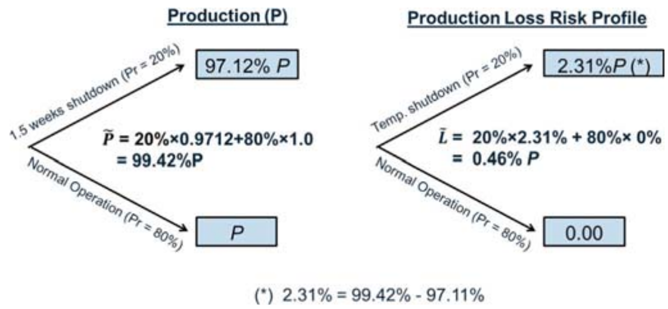


**Fig. C.4.** Temporary Shutdown Risk Profile (incorporating climate change effect).

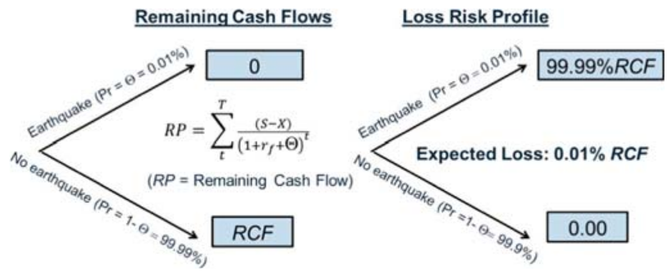


**Fig. C.5.** Permanent Shutdown Risk Profile.

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